CHAPTER 9

PROBABILITY

LEARNING OBJECTIVES

Upon completion of this chapter, you should be able to do the following:

1. Apply the basic concepts of probability.

2. Solve for probabilities of success and failure.

3. Interpret numerical and mathematical expectation.

4. Apply the concept of compound probabilities to independent, dependent, and mutually exclusive events.

5. Apply the concept of empirical events.

INTRODUCTION

The history of probability theory dates back to the 17th century and at that time was related to games of chance. In the 18th century the probability theory was known to have applications beyond the scope of games of chance. Some of the applications in which probability theory is applied are situations with outcomes such as life or death and boy or girl. Statistics and probability are currently applied to insurance, annuities, biology, and social investigations.

The treatment of probability in this chapter is limited to simple applications. These applications will be, to a large extent, based on games of chance, which lend themselves to an understanding of basic ideas of probability.

BASIC CONCEPTS

If a coin were tossed, the chance it would land heads up is just as likely as the chance it would land tails up; that is, the coin has no more reason to land heads up than it has to land tails up. Every toss of the coin is called a *trial.*

We define *probability* as the ratio of the different number of ways a trial can succeed (or fail) to the total number of ways in which it may result. We will let p represent the *probability of success* and q represent the *probability of failure.*

One commonly misunderstood concept of probability is the effect prior trials have on a single trial. That is, after a coin has been tossed many times and every trial resulted in the coin falling heads up, will the next toss of the coin result in tails up? The answer is "not necessarily" and will be explained later in this chapter.

PROBABILITY OF SUCCESS

If a trial must result in any of n equally likely ways, and if s is the number of successful ways and f is the number of failing ways, the probability of success is

$$p = \frac{s}{s + f}$$

where

$$s + f = n$$

EXAMPLE: What is the probability that a coin will land heads up?

SOLUTION: There is only one way the coin can land heads up; therefore, s equals 1. There is also only one way the coin can land other than heads up; therefore, f equals 1. Since

$$s = 1$$

and

$$f = 1$$

then the probability of success is

$$p = \frac{s}{s + f}$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$

This, then, is the ratio of successful ways in which the trial can succeed to the total number of ways the trial can result.

EXAMPLE: What is the probability that a die (singular of dice) will land with a 3 showing on the upper face?

SOLUTION: A die has a total of 6 sides. Therefore the die can land with a 3 face up 1 favorable way and 5 unfavorable ways.

Since

$$s = 1$$

and

$$f = 5$$

then

$$p = \frac{s}{s + f}$$

$$= \frac{1}{1 + 5}$$

$$= \frac{1}{6}$$

EXAMPLE: What is the probability of drawing a black marble from a box of marbles if all 6 of the marbles in the box are white?

SOLUTION: There are no favorable ways of success and there are 6 total ways. Therefore,

$$s = 0$$

and

$$f = 6$$

so that

$$p = \frac{0}{0 + 6}$$

$$= \frac{0}{6}$$

$$= 0$$

EXAMPLE: What is the probability of drawing a black marble from a box of 6 black marbles?

SOLUTION: There are 6 successful ways and no unsuccessful ways of drawing the marble. Therefore,

$$s = 6$$

and

$$f = 0$$

so that

$$p = \frac{6}{6 + 0}$$

$$= \frac{6}{6}$$

$$= 1$$

The previous two examples are the extremes of probabilities and intuitively demonstrate that the probability of an event ranges from 0 to 1 inclusively.

EXAMPLE: A box contains 6 hard lead pencils and 12 soft lead pencils. What is the probability of drawing a soft lead pencil from the box?

SOLUTION: We are given

$$s = 12$$

and

$$f = 6$$

therefore,

$$p = \frac{12}{12 + 6}$$

$$= \frac{12}{18}$$

$$= \frac{2}{3}$$

PRACTICE PROBLEMS:

1. What is the probability of drawing an ace from a standard deck of 52 playing cards?

2. What is the probability of drawing a black ace from a standard deck of playing cards?

3. If a die is rolled, what is the probability of an odd number showing on the upper face?

4. A man has 3 nickels, 2 dimes, and 4 quarters in his pocket. If he draws a single coin from his pocket, what is the probability that

   a. he will draw a nickel?

   b. he will draw a half-dollar?

   c. he will draw a quarter?

ANSWERS:

1. $\frac{1}{13}$

2. $\frac{1}{26}$

3. $\frac{1}{2}$

4. a. $\frac{1}{3}$

   b. 0

   c. $\frac{4}{9}$

PROBABILITY OF FAILURE

As before, if a trial results in any of n equally likely ways, and s is the number of successful ways and f is the number of failures, the probability of failure is

$$q = \frac{f}{s + f}$$

or

$$= \frac{n - s}{n}$$

where

$$s + f = n$$

or

$$n - s = f$$

A trial must result in either success or failure. If success is certain then p equals 1 and q equals 0. If success is impossible then p equals 0 and q equals 1. Combining both events, for either case, makes the probability of success plus the probability of failure equal to 1. If

$$p = \frac{s}{s + f}$$

and

$$q = \frac{f}{s + f}$$

then

$$p + q = \frac{s}{s + f} + \frac{f}{s + f}$$

$$= 1$$

If, in any event

$$p + q = 1$$

then

$$q = 1 - p$$

In the case of tossing a coin, the probability of success is

$$p = \frac{s}{s+f}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

and the probability of failure is

$$q = 1 - p$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

EXAMPLE: What is the probability of not drawing a black marble from a box containing 6 white, 3 red, and 2 black marbles?

SOLUTION: The probability of drawing a black marble from the box is

$$p = \frac{s}{s+f}$$

$$= \frac{2}{2+9}$$

$$= \frac{2}{11}$$

Since the probability of drawing a marble is 1, then the probability of not drawing a black marble is

$$q = 1 - p$$

$$= 1 - \frac{2}{11}$$

$$= \frac{9}{11}$$

PRACTICE PROBLEMS:

Compare the following problems and answers with the preceding problems dealing with the probability of success:

1. What is the probability of not drawing an ace from a standard deck of 52 playing cards?
2. What is the probability of not drawing a black ace from a standard deck of playing cards?
3. If a die is rolled, what is the probability of an odd number not showing on the upper face?
4. A man has 3 nickels, 2 dimes, and 4 quarters in his pocket. If he draws a single coin from his pocket, what is the probability that
   a. he will not draw a nickel?
   b. he will not draw a half-dollar?
   c. he will not draw a quarter?

ANSWERS:

1. $\frac{12}{13}$
2. $\frac{25}{26}$
3. $\frac{1}{2}$
4. a. $\frac{2}{3}$
   b. 1
   c. $\frac{5}{9}$

EXPECTATION

Expectation is the average of the values you would get in conducting an experiment or trial exactly the same way many times. In this discussion of expectation, we will consider two types. One is a numerical expectation and the other is a mathematical expectation.

Numerical Expectation

If you tossed a coin 50 times, you would expect the coin to fall heads (on the average) about 25 times. Your assumption is explained by the following definition of *numerical expectation: If the probability of success in one trial is p, and k is the total number of trials, then kp is the expected number of successes in the k trials.*

In the above example of tossing the coin 50 times, the probability of heads (successes) is

$$E_n = kp$$

where

$$E_n = \text{expected number}$$

$$k = \text{number of tosses}$$

$$p = \text{probability of heads (successes)}$$

Substituting values in the equation, we find that

$$E_n = 50\left(\frac{1}{2}\right)$$

$$= 25$$

EXAMPLE: A die is rolled by a player. What is the expectation of rolling a 6 in 30 trials?

SOLUTION: The probability of rolling a 6 in 1 trial is

$$p = \frac{1}{6}$$

and the number of rolls is

$$k = 30$$

therefore,

$$E_n = kp$$

$$= 30\left(\frac{1}{6}\right)$$

$$= 5$$

In words, the player would expect (on the average) to roll a 6 five times in 30 rolls.

Mathematical Expectation

We will define *mathematical expectation* as follows: *If, in the event of a successful result, amount a, is to be received and the probability of success of that event is p, then ap is the mathematical expectation.*

If you were to buy 1 of 500 raffle tickets for a video recorder worth \$325.00, what would be your mathematical expectation?

In this case, the product of the amount you stand to win and the probability of winning is

$$E_m = ap$$

where

$$a = \text{amount you stand to win}$$

$$p = \text{probability of success}$$

and

$$E_m = \text{expected amount}$$

Then, by substitution

$$\begin{aligned} E_m &= ap \\ &= \$325.00\left(\frac{1}{500}\right) \\ &= \$0.65 \end{aligned}$$

Thus, you would not want to pay more than 65 cents for the ticket, unless, of course the raffle were for a worthy cause.

EXAMPLE: To entice the public to invest in their development, Sunshine Condominiums has offered a prize of \$2,000 to 1 randomly selected family out of the first 1,000 families that participate in the condominium's tour.

1. What would be each family's mathematical expectation?

2. Would it be worthwhile for the Jones family to spend \$3.00 in gasoline to drive to Sunshine Condominiums to take the tour?

SOLUTION:

1. $E_m = ap = \$2{,}000\left(\frac{1}{1{,}000}\right) = \$2.00$

2. No; since \$3.00 is \$1.00 over their expectation of \$2.00, it would not be worthwhile for the Jones family to take the tour.

PRACTICE PROBLEMS:

1. A box contains 7 slips of paper, each numbered differently. A girl makes a total of 50 draws, returning the drawn slip after each draw.

   a. What is the probability of drawing a selected numbered slip in 1 drawing?

   b. How many times would the girl expect to draw the single selected numbered slip in the 50 draws?

2. In a winner-take-all tournament among four professional tennis players, the prize money is \$500,000. Joe Conners, one of the tennis players, figures his probability of winning is 0.20.

   a. What is his mathematical expectation?

   b. Would he be better off if he made a secret agreement with the other tennis players to divide the prize money evenly regardless of who wins?

ANSWERS:

1. a. $\frac{1}{7}$

   b. $7\frac{1}{7}$

2. a. \$100,000

   b. Yes; he would be better off, since he would make \$125,000, which is greater than his expectation of \$100,000.

COMPOUND PROBABILITIES

The probabilities to this point have been single events. In the discussion on compound probabilities, events that may affect others will be covered. The word *may* is used because independent events are included with dependent events and mutually exclusive events.

INDEPENDENT EVENTS

Two or more events are *independent* if the occurrence or nonoccurrence of one of the events has no affect on the probability of occurrence of any of the others.

When two coins are tossed at the same time or one after the other, whether one falls heads or tails has no affect on the way the second coin falls. Suppose we call the coins A and B. The coins may fall in the following four ways:

1. A and B may fall heads.
2. A and B may fall tails.
3. A may fall heads and B may fall tails.
4. A may fall tails and B may fall heads.

The probability of any one way for the coins to fall is calculated as follows:

$$s = 1$$

and

$$n = 4$$

therefore,

$$p = \frac{1}{4}$$

This probability may be determined by considering the product of the separate probabilities; that is,

the probability that A will fall heads is $\frac{1}{2}$

the probability that B will fall heads is $\frac{1}{2}$

and the probability that both will fall heads is

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

In other words, when two events are independent, the probability that one and then the other will occur is the product of their separate probabilities.

EXAMPLE: A box contains 3 red marbles and 7 green marbles. If a marble is drawn, then replaced, and another marble is drawn, what is the probability that both marbles are red?

SOLUTION: Two solutions are offered. First, by the principle of choice, 2 marbles can be selected in $10 \cdot 10$ ways. The red marble may be selected on the first draw in three ways and on the second draw in three ways; and by the principle of choice, a red marble may be drawn on both trials in $3 \cdot 3$ ways. Then the required probability is

$$p = \frac{9}{100}$$

The second solution, using the product of independent events, follows: The probability of drawing a red marble on the first draw is $\frac{3}{10}$, and the probability of drawing a red marble on the second draw is $\frac{3}{10}$. Therefore, the probability of drawing a red marble on both draws is the product of the separate probabilities

or

$$p = \frac{3}{10} \cdot \frac{3}{10} = \frac{9}{100}$$

PRACTICE PROBLEMS:

1. If a die is tossed twice, what is the probability of rolling a 2 followed by a 3?

2. A box contains 2 white, 3 red, and 4 blue marbles. If after each selection the marble is replaced, what is the probability of drawing, in order

   a. a white then a blue marble?

   b. a blue then a red marble?

   c. a white, a red, then a blue marble?

ANSWERS:

1. $\frac{1}{36}$

2. a. $\frac{8}{81}$

   b. $\frac{4}{27}$

   c. $\frac{8}{243}$

DEPENDENT EVENTS

In some cases one event is dependent on another; that is, two or more events are said to be *dependent* if the occurrence or nonoccurrence of one of the events affects the probabilities of occurrence of any of the others.

Consider that two or more events are dependent. If p_1 is the probability of a first event; p_2 the probability that after the first happens, the second will occur; p_3 the probability that after the first and second have happened, the third will occur; etc., then the probability that all events will happen in the given order is the product $p_1 \cdot p_2 \cdot p_3 \cdot \cdot \cdot$.

EXAMPLE: A box contains 3 white marbles and 4 black marbles. What is the probability of drawing 2 black marbles and 1 white marble in succession without replacement?

SOLUTION: On the first draw the probability of drawing a black marble is

$$p_1 = \frac{4}{7}$$

On the second draw the probability of drawing a black marble is

$$p_2 = \frac{3}{6}$$

$$= \frac{1}{2}$$

On the third draw the probability of drawing a white marble is

$$p_3 = \frac{3}{5}$$

Therefore, the probability of drawing 2 black marbles and 1 white marble is

$$\begin{aligned} p &= p_1 \cdot p_2 \cdot p_3 \\ &= \frac{4}{7} \cdot \frac{1}{2} \cdot \frac{3}{5} \\ &= \frac{6}{35} \end{aligned}$$

EXAMPLE: Slips numbered 1 through 9 are placed in a box. If 2 slips are drawn, without replacement, what is the probability that

1. both are odd?

2. both are even?

SOLUTION:

1. The probability that the first is odd is

$$p_1 = \frac{5}{9}$$

and the probability that the second is odd is

$$p_2 = \frac{4}{8}$$

Therefore, the probability that both are odd is

$$\begin{aligned} p &= p_1 \cdot p_2 \\ &= \frac{5}{9} \cdot \frac{4}{8} \\ &= \frac{5}{18} \end{aligned}$$

2. The probability that the first is even is

$$p_1 = \frac{4}{9}$$

and the probability that the second is even is

$$p_2 = \frac{3}{8}$$

Therefore, the probability that both are even is

$$p = p_1 \cdot p_2$$

$$= \frac{4}{9} \cdot \frac{3}{8}$$

$$= \frac{1}{6}$$

A second method of solution involves the use of combinations.

1. A total of 9 slips are taken 2 at a time and 5 odd slips are taken 2 at a time; therefore,

$$p = \frac{{}_5C_2}{{}_9C_2}$$

$$= \frac{5}{18}$$

2. A total of ${}_9C_2$ choices and 4 even slips are taken 2 at a time; therefore,

$$p = \frac{{}_4C_2}{{}_9C_2}$$

$$= \frac{1}{6}$$

PRACTICE PROBLEMS:

In the following problems assume that no replacement is made after each selection:

1. A box contains 5 white and 6 red marbles. What is the probability of successfully drawing, in order, a red marble and then a white marble?

2. A bag contains 3 red, 2 white, and 6 blue marbles. What is the probability of drawing, in order, 2 red, 1 blue, and 2 white marbles?

3. Fifteen airmen are in the line crew. They must take care of the coffee mess and line shack cleanup. They put slips numbered 1 through 15 in a hat and decide that anyone who draws a number divisible by 5 will be assigned the coffee mess and anyone who draws a number divisible by 4 will be assigned cleanup. The first person draws a 4, the second a 3, and the third an 11. What is the probability that the fourth person to draw will be assigned

   a. the coffee mess?

   b. the cleanup?

ANSWERS:

1. $\frac{3}{11}$

2. $\frac{1}{770}$

3. a. $\frac{1}{4}$

   b. $\frac{1}{6}$

MUTUALLY EXCLUSIVE EVENTS

Two or more events are called *mutually exclusive* if the occurrence of any one of them precludes the occurrence of any of the others. The probability of occurrence of two or more mutually exclusive events is the sum of the probabilities of the individual events.

Sometimes when one event has occurred, the probability of another event is excluded (referring to the same given occasion or trial).

For example, throwing a die once can yield a 5 or 6, but not both, in the same toss. The probability that either a 5 or 6 occurs is the sum of their individual probabilities.

$$p = p_1 + p_2$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{1}{3}$$

EXAMPLE: From a bag containing 5 white balls, 2 black balls, and 11 red balls, 1 ball is drawn. What is the probability that it is either black or red?

SOLUTION: The draw can be made in 18 ways. The choices are 2 black balls and 11 red balls, which are favorable, or a total of 13 favorable choices. Then, the probability of success is

$$p = \frac{13}{18}$$

Since drawing a red ball excludes the drawing of a black ball, and vice versa, the two events are mutually exclusive; so the probability of drawing a black ball is

$$p_1 = \frac{2}{18}$$

and the probability of drawing a red ball is

$$p_2 = \frac{11}{18}$$

Therefore, the probability of success is

$$p = p_1 + p_2$$

$$= \frac{2}{18} + \frac{11}{18} = \frac{13}{18}$$

EXAMPLE: What is the probability of drawing either a king, a queen, or a jack from a deck of playing cards?

SOLUTION: The individual probabilities are

$$\text{king} = \frac{4}{52}$$

$$\text{queen} = \frac{4}{52}$$

$$\text{jack} = \frac{4}{52}$$

Therefore, the probability of success is

$$p = \frac{4}{52} + \frac{4}{52} + \frac{4}{52}$$

$$= \frac{12}{52}$$

$$= \frac{3}{13}$$

EXAMPLE: What is the probability of rolling a die twice and having a 5 and then a 3 show or having a 2 and then a 4 show?

SOLUTION: The probability of having a 5 and then a 3 show is

$$p_1 = \frac{1}{6} \cdot \frac{1}{6}$$

$$= \frac{1}{36}$$

and the probability of having a 2 and then a 4 show is

$$p_2 = \frac{1}{6} \cdot \frac{1}{6}$$

$$= \frac{1}{36}$$

Then, the probability of either p_1 or p_2 is

$$p = p_1 + p_2$$

$$= \frac{1}{36} + \frac{1}{36}$$

$$= \frac{1}{18}$$

PRACTICE PROBLEMS:

1. When tossing a coin, you have what probability of getting either a head or a tail?

2. A bag contains 12 blue, 3 red, and 4 white marbles. What is the probability of drawing

   a. in 1 draw, either a red or a white marble?

   b. in 1 draw, either a red, white, or blue marble?

   c. in 2 draws, either a red marble followed by a blue marble or a red marble followed by a red marble?

3. What is the probability of getting a total of at least 10 points in rolling two dice? (HINT: You want either a total of 10, 11, or 12.)

ANSWERS:

1. 1

2. a. $\frac{7}{19}$

   b. 1

   c. $\frac{7}{57}$

3. $\frac{1}{6}$

EMPIRICAL PROBABILITIES

Among the most important applications of probability are those situations where we cannot list all possible outcomes. To this point, we have considered problems in which the probabilities could be obtained from situations of equally likely results.

Because some problems are so complicated for analysis, we can only estimate probabilities from experience and observation. This is *empirical probability*.

In modern industry probability now plays an important role in many activities. Quality control and reliability of a manufactured article have become extremely important considerations in which probability is used.

Experience has shown that empirical probabilities, if carefully determined on the basis of adequate statistical samples, can be applied to large groups with the result that probability and relative frequency are approximately equal. By adequate samples we mean a large enough sample so that accidental runs of "luck," both good and bad, cancel each other. With enough trials, predicted results and actual results agree quite closely. On the other hand, applying a probability ratio to a single individual event is virtually meaningless.

Table 9-1.—Weather Forecast

Date	Forecast	Actual weather	Did the actual forecasted event occur?
1	Rain	Rain	Yes
2	Light showers	Sunny	No
3	Cloudy	Cloudy	Yes
4	Clear	Clear	Yes
5	Scattered showers	Warm and sunny	No
6	Scattered showers	Scattered showers	Yes
7	Windy and cloudy	Windy and cloudy	Yes
8	Thundershowers	Thundershowers	Yes
9	Clear	Cloudy and rain	No
10	Clear	Clear	Yes

We define *relative frequency of success* as follows: *After N trials of an event have been made, of which S trials are successes, the relative frequency of success is*

$$P = \frac{S}{N}$$

For example, table 9-1 shows a small number of weather forecasts from April 1st to April 10th. The actual weather on the dates is also given.

Observe that the forecasts on April 1, 3, 4, 6, 7, 8, and 10 were correct. We have observed 10 outcomes. The event of a correct forecast has occurred 7 times. Based on this information we might say that the probability for future forecasts being true is 7/10. This number is the best estimate we can make from the given information. In this case, since we have observed such a small number of outcomes, we would be incorrect to say that the estimate of P is dependable. A great many more cases should be used if we expect to make a good estimate of the probability that a weather forecast will be accurate. A great many factors affect the accuracy of a weather forecast. This example merely indicates something about how successful a particular weather office has been in making weather forecasts.

Another example may be drawn from industry. Many thousands of articles of a certain type are manufactured. The company selects 100 of these articles at random and subjects them to very careful tests. In these tests 98 of the articles are found to meet all measurement requirements and perform satisfactorily. This suggests that 98/100 is a measure of the reliability of the article.

One might expect that about 98% of all of the articles manufactured by this process will be satisfactory. The probability (measure of chance) that one of these articles will be satisfactory might be said to be 0.98.

This second example of empirical probability is different from the first example in one very important respect. In the first example we could list all of the possibilities, and in the second example we could not do so. The selection of a sample and its size is a problem of statistics.

Considered from another point of view, statistical probability can be regarded as relative frequency.

EXAMPLE: In a dart game, a player hit the bull's eye 3 times out of 25 trials. What is the statistical probability that he will hit the bull's eye on the next throw?

SOLUTION:

$$N = 25$$

and

$$S = 3$$

hence

$$P = \frac{3}{25}$$

EXAMPLE: Using table 9-2, what is the probability that a person 20 years old will live to be 50 years old?

SOLUTION: Of 95,148 persons at age 20, 81,090 survived to age 50. Hence

$$P = \frac{81,090}{95,148}$$

$$= 0.852 \text{ (rounded)}$$

EXAMPLE: How many times would a die be expected to land with a 5 or 6 showing in 20 trials?

Table 9-2.—Mortality Table (Based on 100,000 Individuals 1 Year of Age)

Age	Number of people
5	98,382
10	97,180
15	96,227
20	95,148
25	93,920
30	92,461
35	90,655
40	88,334
45	85,255
50	81,090
55	75,419
60	67,777
65	57,778
70	45,455
75	31,598
80	18,177
85	7,822
90	2,158

SOLUTION: The probability of a 5 or 6 showing is

$$p = \frac{1}{3}$$

The relative frequency is approximately equal to the probability

$$P \approx p$$

Therefore, since

$$P = \frac{S}{N}$$

where

$$P = \frac{1}{3}$$

$$N = 20$$

$$S = ?$$

then rearranging and substituting, we find that

$$\begin{aligned} S &= NP \\ &= 20\left(\frac{1}{3}\right) \\ &= \frac{20}{3} \\ &= 6.67 \text{ (rounded)} \end{aligned}$$

This says that the expected number of times a die would land with a 5 or 6 showing in 20 trials is 6.67; that is, on the average a die will land with a 5 or 6 showing 6.67 times per 20 trials.

PRACTICE PROBLEMS:

1. A construction crew consists of 6 electricians and 38 other workers. How many electricians would you expect to choose if you choose 1 person each day of a workweek for your

helper? (Sunday will not be considered part of the workweek.)

2. How many times would a tossed die be expected to turn up a 3 or less in 30 tosses?

3. Using table 9-2, find the probability that a person whose age is 30 will live to age 60.

ANSWERS:

1. 0.82

2. 15

3. 0.733

SUMMARY

The following are the major topics covered in this chapter:

1. **Probability:** *Probability* is the ratio of the different number of ways a trial can succeed (or fail) to the total number of ways in which it may result.

2. **Probabilities of success and failure:** If a trial must result in any of n equally likely ways, and if s is the number of successful ways and f is the number of failing ways, the *probability of success* is

$$p = \frac{s}{s + f}$$

   and the *probability of failure* is

$$q = \frac{f}{s + f} \text{ or } q = \frac{n - s}{n}$$

   where $s + f = n$ or $n - s = f$

3. **Expectation:** *Expectation* is the average of the values you would get in conducting an experiment or trial exactly the same way many times.

4. **Numerical expectation:** If the probability of success in one trial is p, and k is the total number of trials, then kp is the expected number of successes in the k trials or

$$E_n = kp$$

5. **Mathematical expectation:** If, in the event of a successful result, amount a is to be received, and p is the probability of success of that event, then ap is the mathematical expectation or

$$E_m = ap$$

6. **Independent events:** Two or more events are *independent* if the occurrence or nonoccurrence of one of the events has no effect on the probability of occurrence of any of the others.

7. **Dependent events:** Two or more events are *dependent* if the occurrence or nonoccurrence of one of the events affects the probabilities of occurrence of any of the others.

8. **Mutually exclusive events:** Two or more events are called *mutually exclusive* if the occurrence of any one of them precludes the occurrence of any of the others.

9. **Empirical probability:** *Empirical probability* is an estimated probability from experience and observation.

10. **Relative frequency of success:** After N trials of an event have been made, of which S trials are successes, the relative frequency of success is

$$P = \frac{S}{N}$$

ADDITIONAL PRACTICE PROBLEMS

1. A box contains 5 red marbles, 6 blue marbles, and 7 green marbles. If 1 marble is to be drawn, what is the probability that it is

   a. green?

   b. red?

   c. yellow?

2. A box contains 5 red marbles, 6 blue marbles, and 7 green marbles. If 1 marble is to be drawn, what is the probability that it is not

   a. green?

   b. red?

   c. yellow?

3. A child is to pick a letter of the alphabet from a box.

   a. What is the child's probability of picking a vowel in 1 draw? (*Y* will not be considered as a vowel.)

   b. What is the child's numerical expectation of picking a vowel in 20 draws? (The letter will be replaced after each draw.)

4. A concert promoter agrees to pay a band \$5,600 in case the concert has to be cancelled because of rain. The promoter's actuary figures expected loss for this risk to be \$717. What probability is assigned to the possibility that the concert will have to be cancelled because of rain?

5. A basket contains 3 apples, 5 pears, and 7 oranges. If after each selection the fruit is replaced, what is the probability of drawing, in order,

   a. an orange, then a pear?

   b. 2 apples?

   c. an apple, an orange, then a pear?

6. A car has 8 spark plugs, of which 3 are defective. Find the probability of locating all 3 defective spark plugs in 3 selections, without replacement.

7. In a convention of 120 politicians, 52 are Democrats and 33 are Republicans. Find the probability that a politician selected is a Democrat or a Republican.

8. Routine medical examinations are given to 44 smokers and 62 nonsmokers. If one of the subjects is selected for more detailed tests, what is the probability that the selected subject smokes?

9. In a classroom of 33 girls and 22 boys, how many girls would you expect to choose in 12 trials?

ANSWERS TO ADDITIONAL PRACTICE PROBLEMS

1. a. 7/18

   b. 5/18

   c. 0

2. a. 11/18

   b. 13/18

   c. 1

3. a. 5/26

   b. 3.85

4. 0.128

5. a. 7/45

   b. 1/25

   c. 7/225

6. 1/56

7. 17/24

8. 22/53

9. 7.2